1. (a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

Tips : if you don't know where to start, always consider putting values into the equation.

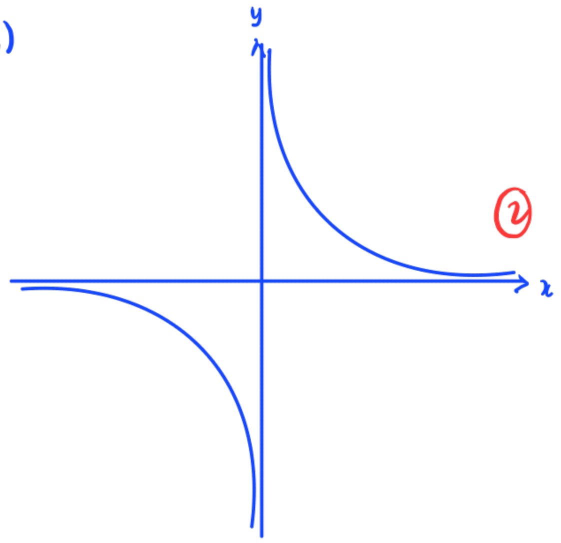
(2)

(b) Hence or otherwise, solve

$$\frac{16}{x} \leqslant 2$$

(3)

a)



if $x = 0,\ y = \infty$

$y = 0,\ x = \infty$

hence, the curve should not touch both x and y axis.

b) $\frac{16}{x} = 2$, $x < 0$ 1

substitute into the equation, when $x < 0$, the value on RHS will always be ≤ 2. So, $x < 0$ is a solution.

$x = \frac{16}{2} = 8$

x must be ≥ 8 because $x < 0$ is already considered above.

Hence,

$\therefore\ x < 0\ ,\ x \geq 8$ 1

2. The point $P(-2, -5)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$ **(1)**

(b) $y = |f(x)|$ **(1)**

(c) $y = 3f(x - 2) + 2$ **(2)**

(a) $y = f(x) + 2$ $f(x) = -5$ from question

$y = -5 + 2$

$y = -3$ the x value is not changed

P becomes $(-2, -3)$ 1

(b) $y = |f(x)|$

$y = |-5|$ ← $|a|$ takes the magnitude of a

$y = 5$

P becomes $(-2, 5)$ 1

(c) $y = 3f(x-2) + 2$

$x = -2$ ← $x - a$ changes the x-value by $+a$

$x' = -2 + 2$

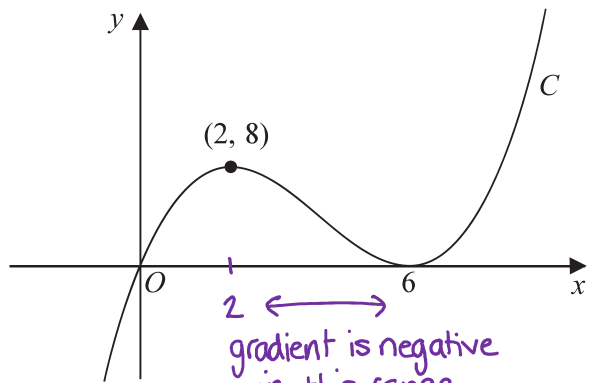
$x' = 0$ 1

$y' = 3(-5) + 2$

$y' = -13$

P becomes $(0, -13)$ 1

3.



2 ⟵⟶ gradient is negative in this range

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

(a) $2 < x < 6$ (1)

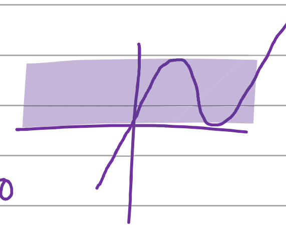
$\mathrm{f}'(x) < 0$ means the gradient is negative.
Negative gradient = line going down. ↘

(b) $k > 8$ or $k < 0$ (1)

$\{k : k > 8\} \cup \{k : k < 0\}$ (1)

$y = k$ is a horizontal line through the y-axis.

has to be outside of shaded region to intersect only once. →



CHOOSE ONE OF THESE METHODS.

(c) Method 1 : Recognise curve has form $y = ax(x-6)^2$ ① states form of c

$(2,8) \rightarrow \quad 8 = 2a(2-6)^2$ ①

$8 = 32a$

$a = \frac{1}{4}$

$\therefore y = \frac{1}{4}x(x-6)^2$ ①

Method 2 : Solving Simultaneous Equations

$y = ax^3 + bx^2 + cx$ ← no $+d$ because the curve goes through the origin.

when $x = 2$, $y = 8$:

$8 = a(2^3) + b(2^2) + c(2)$

① $4 = 4a + 2b + c$

when $x = 6$, $y = 0$:

$0 = a(6^3) + b(6^2) + c(6)$

② $0 = 216a + 36b + 6c$ ① for 2 sim. eq.

$f'(x) = 3ax^2 + 2bx + c$

when $x = 6$, $f'(x) = 0$: ← $(6,0)$ is a turning point

$0 = 3a(6^2) + 2b(6) + c$

③ $0 = 108a + 12b + c$

Solve ①, ②, ③ simultaneously : ← use a calculator or solve by hand.

$4 = 4a + 2b + c$

$0 = 216a + 36b + 6c$

$0 = 108a + 12b + c$

$a = \frac{1}{4}, \quad b = -3, \quad c = 9$ ① for solving sim. eq.

$y = \frac{1}{4}x^3 - 3x^2 + 9x$ ①

4.

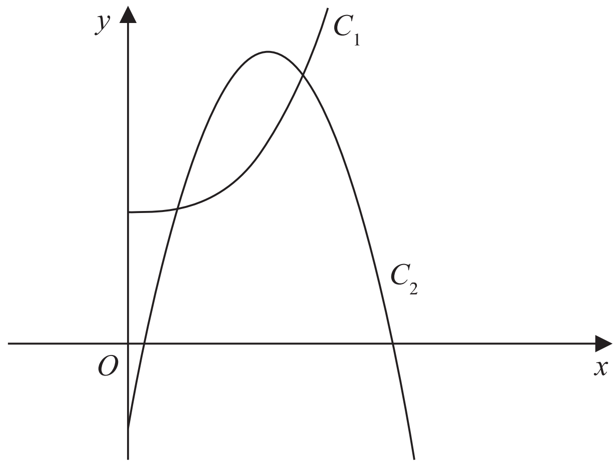


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

(a) when $x = \frac{1}{2}$:

C_1: $y = 2(\frac{1}{2})^3 + 10$

$= \frac{41}{4}$

C_2: $y = 42(\frac{1}{2}) - 15(\frac{1}{2})^2 - 7$ (1)

$= \frac{41}{4}$

$\therefore$ C_1 and C_2 intersect at $(\frac{1}{2}, \frac{41}{4})$ (1)

(b) $2x^3 + 10 = 42x - 15x^2 - 7$ (1)

$2x^3 + 15x^2 - 42x + 17 = 0$

$2x - 1$ is a factor of this equation - this could be deduced by inspection, trial-and-error or any other valid method.

$$\begin{array}{r|l} & x^2 + 8x - 17 \\ \hline 2x-1 & 2x^3 + 15x^2 - 42x + 17 \\ & 2x^3 - x^2 \\ \hline & 0 + 16x^2 \\ & 16x^2 - 8x \\ \hline & 0 - 34x \\ & -34x + 17 \\ \hline & 0 + 0 \end{array}$$ (1)

you don't have to do long division - inspection or other valid algebraic methods are accepted.

$2x^3 + 15x^2 - 42x + 17 = 0 \Rightarrow (2x-1)(x^2 + 8x - 17) = 0$ (1)

$2x - 1 = 0 \Rightarrow x_1 = \frac{1}{2}$

solve $x^2 + 8x - 17$ using a calculator, or the quadratic equation.

from $x^2 + 8x - 17$:

$x_2 = -4 + \sqrt{33}$

$x_3 = -4 - \sqrt{33}$ (1) ($x = \frac{1}{2}$ is the other intercept)

The point P is on the positive side of the y-axis, therefore:

$x = -4 + \sqrt{33}$ (1)

5. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(2)

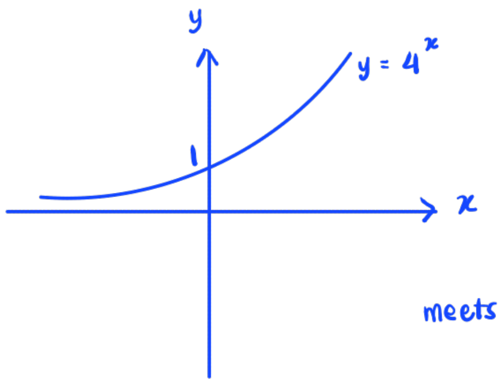
(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

a)



2

meets y-axis at (0,1)

b) $4^x = 100$

$\ln(4^x) = \ln(100)$

$x \ln(4) = \ln(100)$

$x = \dfrac{\ln(100)}{\ln(4)} = 3.321928$ 1

$x = 3.32$ (2 d.p.)

1